## EXPLORE ROBOTICS CISC 1003

## DEGREES OF FREEDOM



## REVIEW LOCOMOTION AND DEGREES OF FREEDOM

## Locomotion

- Locomotion = locus (place) + motion
- Locomotion refers to the way a body moves
- from place to place.
- A fundamental function of humans, animals
- Acquired through training
- Requiring significant "brain power"
- It's generally the first challenge for a robot
- Many modes of locomotion exist


## Modes of Locomotion

- Most common, legged vs. Wheeled
- Benefits and challenges:
- Wheeled:
- Most efficient use of power
- Low number of Degrees of Freedom (DOFs).
- Legged:
- Large DOFs, challenge of stability.


## Locomotion and Manipulations

- Choice of effectors and actuators sets the limits on what the robot can do
- Usually categorized as locomotion or manipulation
- Locomotion: vehicle moving itself
- Manipulation: An arm moving things
- In both cases can consider the degrees of freedom in the design


## Degrees of Freedom (D.O.F.)

- The number of independent parameters that define a systems' configuration or state


## Degrees of freedom (D.O.F.)

- Degrees of Freedom of a point in space:
- \# of independent coordinates required to define position and orientation
- How many such points?



## Degrees of freedom (D.O.F.)

- Degrees of Freedom of a point in space:
- \# of independent coordinates required to define position and orientation
- How many such coordinates?
- 3 coordinates $-x, y$, and $z$



## Degrees of freedom (D.O.F.)

- Degrees of Freedom of a rigid body in space:
- 3 linear movements across $x, y$ and $z$ dimensions
- Three rotational movements about axis $x, y$ and $z$
- Called roll, pitch, and yaw



## Degrees of freedom (D.O.F.)

- Definition: How many independent factors needed to specify the motion of the system?
- The specific number of axes that a rigid body is able to freely move in three-dimensional space
- For robots: directions of independent motions



## Degrees of freedom (D.O.F.)

- For a rigid object in space have:
- The body can move straight in three dimensions:
- Without rotation
- on the $\boldsymbol{X}, \boldsymbol{Y}$ and $\boldsymbol{Z}$ axes
- A.K.A. Translational degrees of freedom
- Also, it can change orientation between those axes though rotation
- usually called pitch, yaw and roll
- Rotational degrees of freedom
- Total of 6 degrees of freedom



## D.O.F. of a Rigid Body in space



## Degrees of Freedom

- How can we see this?
- Let's say we have a square object



## D.O.F. of a Rigid Object in Space

- We can first set the middle point of the cube
- 3 Degrees of freedom $-\mathbf{x , y}$ and $\mathbf{z}$
- Then, we can rotate the cube around each of the axis
- pitch, yaw and roll



## D.O.F. of a Rigid Body in Space

- Alternatively, we can try to position each of the corners
- How many D.O.F. do we have?


## Degrees of Freedom



## Degrees of Freedom

- Point A can have 3 values ( $x, y, z$ )
- Once point $A$ is set, we want to fix point $B$
- However, the length between $A$ and $B$ is constant
- So only two angles can be fixed
- We have one constrain on the location of $B$
-What is the constraint on B?
- B can be located on a sphere
- The sphere radios is the length between $A$ and $B$


## Degrees of Freedom



## Degrees of Freedom

- Point A can have 3 values ( $x, y, z$ )
- Once point $A$ is set, we want to fix point $B$
- However, the length between $A$ and $B$ is constant
- So only two angles can be fixed
- We have one constrain on the location of B


## Degrees of Freedom

- Once $A$ and $B$ are fixed, only one angle is possible for point $C$
- One additional degree of freedom
- We have two constraints on the location of C
- All points will be on a circle with consant radius
- One D.O.F. = angle



## Degrees of Freedom



## Degrees of Freedom

- How many possibilities for point D?
- Zero D.O.F. - only one possible location


## Degrees of Freedom

- \# of D.O.F. $=\sum$ (Freedom of Points) \# of independent constraints
- Since robot is made of rigid bodies:
- \# of D.O.F. $=\sum$ (Freedom of bodies) \# of independent constraints


## Degrees of Freedom



## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | $?$ | $?$ |  |
| B |  |  |  |
| C |  |  |  |
| D |  |  |  |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | $?$ |
| B |  |  |  |
| C |  |  |  |
| D |  |  |  |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | 3 |
| B | $?$ | $?$ |  |
| C |  |  |  |
| D |  |  |  |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | 3 |
| B | 3 | 1 | $?$ |
| C |  |  |  |
| D |  |  |  |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | 3 |
| B | 3 | 1 | 2 |
| C | $?$ | $?$ |  |
| D |  |  |  |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | 3 |
| B | 3 | 1 | 2 |
| C | 3 | 2 | $?$ |
| D | $?$ | $?$ |  |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | 3 |
| B | 3 | 1 | 2 |
| C | 3 | 2 | 1 |
| D | 3 | 3 | 0 |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | 3 |
| B | 3 | 1 | 2 |
| C | 3 | 2 | 1 |
| D | 3 | 3 | 0 |
| Total Degrees <br> of Freedom |  |  | $?$ |

## Degrees of Freedom

| Point | Coordinates | Indep. <br> constraints | \# Actual <br> freedoms |
| :--- | :--- | :--- | :--- |
| A | 3 | 0 | 3 |
| B | 3 | 1 | 2 |
| C | 3 | 2 | 1 |
| D | 3 | 3 | 0 |
| Total Degrees <br> of Freedom |  |  | 6 |

## Degrees of Freedom

- How many degrees of freedom will be in 4dimensional space?
- 10 degrees of freedom


## Degrees of Freedom

- \# of D.O.F. $=\sum$ (Freedom of Points) \# of independent constraints
- Since robot is made of rigid bodies:
- \# of D.O.F. $=\sum$ (Freedom of bodies) \# of independent constraints


## Degrees of Freedom

- How many degrees of freedom are for an object on a linear space?
- l.e., a car
- Linear space is a 2D space
- 3 degrees of freedom
- 2 on the linear space ( $\mathrm{x}, \mathrm{y}$ )
- One is the angle



## Degrees of Freedom - 2D space



https://www.smlease.com/entries/mechanism/what-is-degree-of-freedom-dof-in-mechanics/ https://www.quora.com/How-many-degrees-of-freedom-of-rigid-body

## Degrees of Freedom



## Degrees of Freedom



- Roll, pitch, yaw:
- Degrees of freedom used for orientation
- Yaw refers to the direction in which the body is facing
- i.e., its orientation within the xy plane
- Roll refers to whether the body is upside-down or not
- i.e., its orientation within the yz plane
- Pitch refers to whether the body is tilted
- i.e., its orientation within the xz plane


## Degrees of Freedom

- Degrees of Freedom
- Example: Degrees of Freedom - 6 axis robot


## Controllable D.O.F.'s

- If the object can move in each direction of the D.O.F., it is Holonomic
- All are controllable
- For this, it needs to have an actuator in each direction


## Degrees of freedom (D.O.F.)

- How many D.O.F. to specify movement of a vehicle on a flat surface?
- Three: $X, Y$ and yaw (turn in $x-y$ dimension)
- How many Controllable D.O.F.'s?
- In which direction can driver drive the car?




## Degrees of freedom (D.O.F.)

- How many D.O.F. to specify movement of a vehicle on a flat surface?
- Three: $X, Y$ and yaw (turn in $x-y$ dimension)
- How many Controllable D.O.F.'s?
- In which direction can driver drive the car?
- X and yaw



## Degrees of freedom (D.O.F.)

- Is a car Holonomic?
- Yes, a car is holonomic
- No, a car is not holonomic
- Depending on the car design



## Degrees of freedom (D.O.F.)

- Is a car Holonomic?
- Yes, a car is holonomic
- No, a car is not holonomic
- Depending on the car design
-Why?
- A car can not drive sideways - not holonomic
- Only controllable in two dimensions


## Degrees of freedom (D.O.F.)

Total Degrees of Freedom

Controllable Degrees of Freedom


## Robot's Variables Affecting D.O.F.

- Number of joints/articulations/moving parts
- If parts are linked, fewer parameters needed to specify them.
- Number of Individually controlled moving part
- Need parameters for each to define configuration
- Often described as 'controllable degrees of freedom'
- But some may be redundent
- Two movements may be in the same axis


## D.O.F. OF A ROBOT

## Degrees of Freedom

- \# of D.O.F. $=\sum$ (Freedom of Points) \# of independent constraints
- Since robot is made of rigid bodies:
- \# of D.O.F. $=\sum$ (Freedom of bodies) \# of independent constraints


## Degrees of Freedom

-Where do the constraints come from?

- For a robot?
- Typically, from joints


## ROBOTS JOINTS

## Robot Joints

- Revolute Joint:
- 5 constraints
- 1 Degree of freedom
- Angle of rotation
- Revolute Joint


## Revolute joint - Degrees of Freedom

## Robot Joints

- Prismatic joint:
- A.K.A. linear joint
- 5 constraints
- 1 Degree of freedom
- Prismatic Joint


## Prismatic joint



- https://www.mathworks.com/help/physmod/sm/mech/ref/cylindrical.html


## Prismatic Joint



- https://fastenerengineering.com/what-is-a-prismatic-joint/


## Robot Joints

- Universal Joint:
- 4 constraints
- 2 degrees of freedom
- Universal joint, Universal joint 2


## Universal Joint

## Universal Joint



## Robot Joints

- Spherical joint:
- 3 constraints
- 3 degrees of freedom
- Spherical Joint


## Spherical Joint



- https://www.researchgate.net/publication/311414943_Research_on_OscillationFree_Robust_Control_for_Active_Joint_Dental_Automation/figures?lo=1


## D.O.F. of Robot Joints

| Joint Type | D.O.F. | \# CONSTRAINTS |
| :--- | :--- | :--- |
| Revolute | $?$ | $?$ |
| Prismatic | $?$ | $?$ |
| Universal | $?$ | $?$ |
| Spherical | $?$ | $?$ |

## D.O.F. of Robot Joints

| Joint Type | D.O.F. | \# CONSTRAINTS |
| :--- | :--- | :--- |
| Revolute | $?$ | 5 |
| Prismatic | $?$ | 5 |
| Universal | $?$ | 4 |
| Spherical | $?$ | 3 |

## D.O.F. of Robot Joints

| Joint Type | D.O.F. | \# CONSTRAINTS |
| :--- | :--- | :--- |
| Revolute | 1 | 5 |
| Prismatic | 1 | 5 |
| Universal | 2 | 4 |
| Spherical | 3 | 3 |

## Robot's D.O.F.

- Total D.O.F. $=\Sigma($ freedom of body parts $)-$ \# of independent constraints
- $N=\#$ of bodies, not including ground
- $J=\#$ of joints
- $m=6$ for spatial bodies, 3 for planar
-D.O.F. $=m * N-\Sigma_{i=1}^{J} c_{i}$


